6. Electromagnetic induction

Electromagnetic Induction (EMI)

The phenomena of generating current and e.m.f. by changing magnetic fields is called Electromagnetic Induction (EMI).

Induced e.m.f. & Induced Current

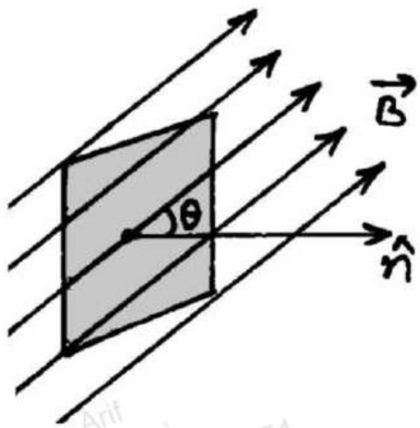
The e.m.f. so developed is called induced e.m.f. If the conductor is in the form of a closed circuit then a current flow in the circuit. This is called induced current.

* The phenomena of EMI is the basis of working of power generators, dynamos, transformers etc.

Magnetic Flux (ϕ)

The magnetic flux (ϕ) through any surface held in a magnetic field $\vec{B}$ is measured by the total number of magnetic lines of force crossing the surface normally.

$$\boxed{\phi = \vec{B}\cdot\vec{A} = BA\cos\theta}$$



If the coil has N turns then total amount of magnetic flux linked with the coil is

$$\boxed{\phi = N(\vec{B}\cdot\vec{A}) = NBA\cos\theta}$$

* when $\theta = 90^\circ$ then $\phi = 0$
* when $\theta = 0^\circ$ then $\phi = BA$ (Max: value)

SI Unit of magnetic flux is weber (Wb).

1 Weber (Wb) -

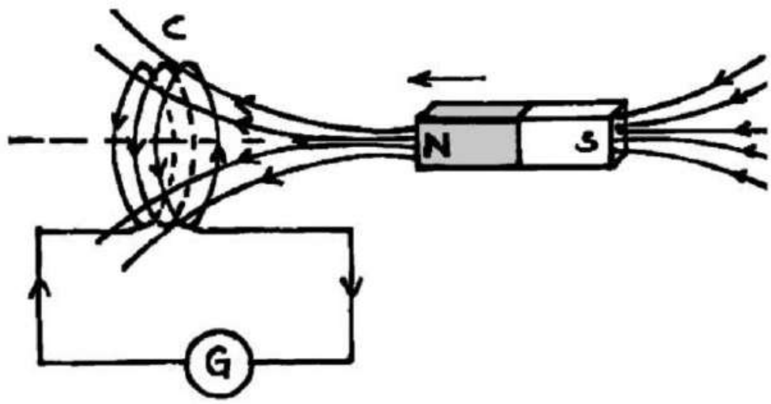
one weber is the amount of magnetic flux over an area of 1 metre^2 held normal to a uniform magnetic field of 1 Tesla.

$$1 \text{ weber} = 1 \text{ tesla} \times 1 \text{ m}^2$$

- * Magnetic flux (ϕ) is a scalar quantity.
- * When a body is present in a uniform or non-uniform magnetic field, outward flux is taken to be positive, while inward flux is taken as negative.

Experiments of Faraday and Henry

Experiment - 1 (Current induced by a magnet)



(i) When north pole of a bar magnet is pushed towards the coil, the galvanometer shows a sudden deflection, which indicates that current is induced in the coil.

(ii) There is no deflection when the bar magnet is held stationary anywhere.

(iii) When the magnet is moved away from the coil, the galvanometer shows deflection in the opposite direction, which indicates reversal in the direction of induced current.

(iv) When south pole of bar magnet is moved towards or away from the coil, the galvanometer deflections are opposite to those observed with the north pole for similar movement.

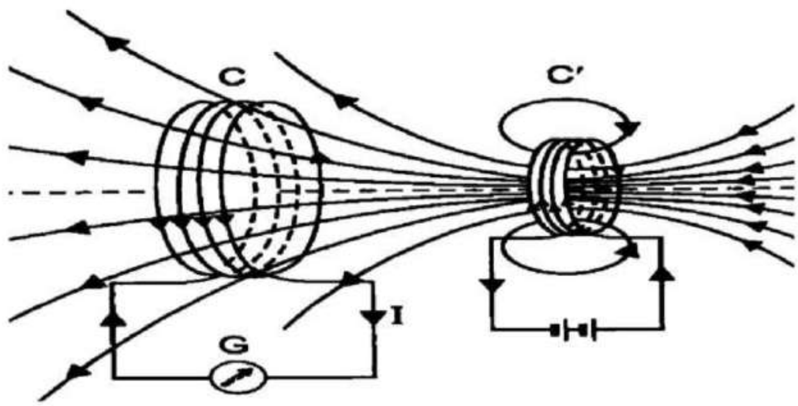
(v) The galvanometer deflection (and hence induced current) is found to be larger when magnet is pushed towards or pulled away from the coil faster.

(vi) When the bar magnet is held stationary and coil C is moved towards or away from the magnet, the same effects are observed.

It shows relative motion between the coil and the magnet is responsible for induction of electric current in the coil.

Experiment 2 - (Current induced by current)

(i) When coil C' is moved towards the coil C, the galvanometer shows a sudden deflection. This indicates that electric current is induced in the coil C.



(ii) When the coil C' is moved away from the coil C, the galvanometer shows a deflection in the opposite direction. This indicates that direction of current induced in coil C is reversed.

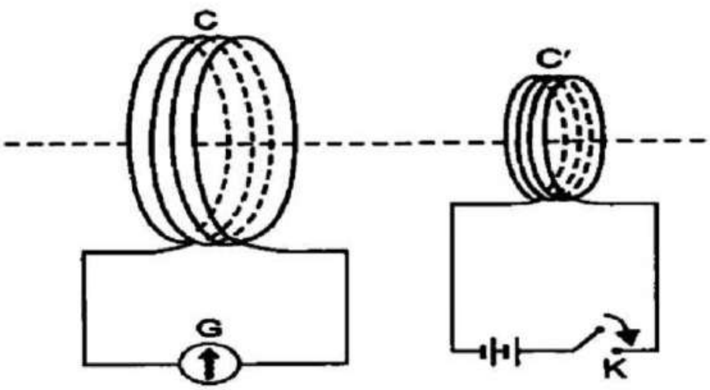
(iii) There is no deflection when there is no relative motion between the two coils.

(iv) The galvanometer deflection (and hence induced current) is found to be larger when coils are moved faster towards / away from each other.

* Here the coil c' carrying current behaves as a bar magnet.

Experiment 3 - (Current induced by Changing Current)

From the experiment 3 Faraday showed that relative motion is not an absolute requirement. Current can be induced without relative motion.



(i) When Key K is pressed, galvanometer in coil c shows a momentary deflection, indicating that current is induced in coil c.

(ii) When the Key is pressed Continuously, there is no deflection in the galvanometer.

(iii) When the Key is released the galvanometer shows again a momentary deflection but in the opposite direction.

(iv) The galvanometer deflection increases, when an iron rod is inserted into the coils along their axis., and the Key is pressed / released.

Cause of Induced E.M.F.

All experiment shows that induced e.m.f. appears in a coil whenever the amount of magnetic flux linked with the coil changes with time.

Faraday's Law of Electromagnetic Induction

First Law -

whenever the amount of magnetic flux linked with a circuit changes, an e.m.f is induced in the circuit. The induced e.m.f. exists so long as the change in magnetic flux continues.

Second Law -

The magnitude of e.m.f. induced in a circuit is directly proportional to the rate of change of magnetic flux linked with the circuit.

$$\text{induced e.m.f} \quad e \propto \frac{(\phi_2 - \phi_1)}{t}$$

or

$$e = K\frac{(\phi_2 - \phi_1)}{t}$$

Here $K = 1$ in all system of Units.

so $\boxed{e = \frac{\phi_2 - \phi_1}{t}}$ Or $\boxed{e = -\frac{d\phi}{dt}}$

* Negative sign is taken because induced e.m.f. always opposes any change in magnetic flux associated with the circuit.

In case of a closely wound coil of N turns,

$$\boxed{e = -N\frac{d\phi}{dt}}$$

So by increasing the number of turns N in the coil we can increase the induced e.m.f.

If the total resistance of the closed loop (Coil) is R then

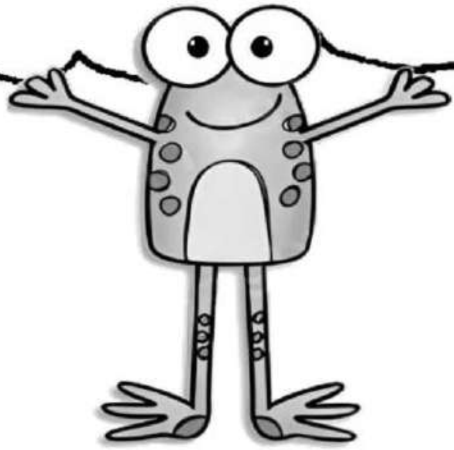
$$\text{Induced current } I = \frac{e}{R} \Rightarrow \boxed{I = -\frac{N}{R}\frac{d\phi}{dt}}$$

The charge induced in the circuit in time Δt is given as

$$\because \quad I = \frac{\Delta Q}{\Delta t} \quad \Rightarrow \quad I = -\frac{N}{R}\frac{\Delta \phi}{\Delta t} = \frac{\Delta Q}{\Delta t}$$

$$\Rightarrow \boxed{\Delta Q = -\frac{N}{R}\Delta \phi}$$

* Induced emf does not depends on nature of the coil and its resistance.
* The induced charge depends on the net change in the magnetic flux and not on the time interval Δt of flux change.
* Induced current and induced charge depends on resistance of the coil (or circuit).



Q. A circular coil of radius 10 cm, 500 turns and resistance $2\,\Omega$ is placed with its plane perpendicular to the horizontal component of the earth's magnetic field. It is rotated about its vertical diameter through 180° in 0.25 s. Estimate the magnitude of the e.m.f. and current induced in the coil. Horizontal component of earth's magnetic Field is 3×10^{-5} T.

Sol. Here - $r = 10\text{ cm} = 10^{-1}\text{ m.}$, $N = 500$

$R = 2\,\Omega$, $\theta_1 = 0^\circ$, $\theta_2 = 180^\circ$

$dt = 0.25\text{ s}$

$$e = -N\frac{(\phi_2 - \phi_1)}{dt} = \frac{-NBA(\cos\theta_2 - \cos\theta_1)}{dt}$$

$$e = \frac{-500\times3\times10^{5}\times3.14\times10^{-2}}{0.25}(\cos180^\circ - \cos0^\circ)$$

$$e = 3.8\times10^{-3}\text{ V}$$

$$I = \frac{e}{R} = \frac{3.8\times10^{-3}}{2} = 1.9\times10^{-3}\text{ A}$$

Q. The magnetic flux through a coil perpendicular to its plane and directed into paper is varying according to the relation $\phi = (5t^2 + 10t + 5)$ mWb. Calculate the e.m.f. induced in the loop at $t = 5$ s.

Sol. Here $\phi = (5t^2 + 10t + 5)$ m Wb

$= (5t^2 + 10t + 5)\times10^{-3}$ Wb.

As $e = -\frac{d\phi}{dt} = -\frac{d}{dt}(5t^2 + 10t + 5)\times10^{-3}$ Wb/s

$e = -(10t + 10)\times10^{-3}$ Volt

At $t = 5$ Sec.

$e = -(10\times5 + 10)\times10^{-3}$ Volt

$e = -0.06$ Volt.

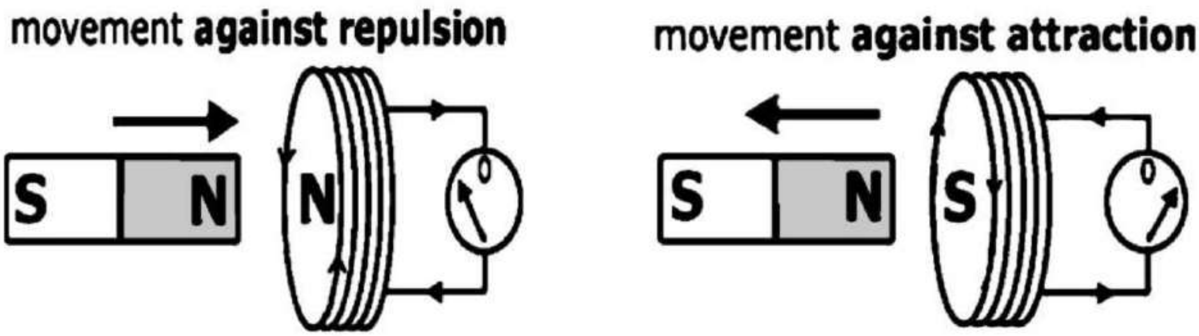
Lenz's Law

According to 'Lenz's Law', the polarity of the induced e.m.f. is such that it opposes the change in the magnetic flux responsible for its production.

This Law gives the direction of current induced in a circuit.

Explanation - When north pole of a bar magnet is being pushed towards the coil, the amount of magnetic flux linked with the coil increases.

Current is induced in the coil in such a direction that it opposes the increase in flux. This is possible only when the current induced in the coil is in anticlockwise direction with respect to an observer on the side of the bar magnet.



Similarly, when north pole of the bar magnet is moved away from the coil the magnetic flux linked with the coil decreases. To oppose this decrease in magnetic flux, current induced in the coil is in clockwise direction so that its face towards the bar magnet becomes south pole.

This would result in an attractive force, which opposes the motion of the magnet and the corresponding decrease in magnetic flux.

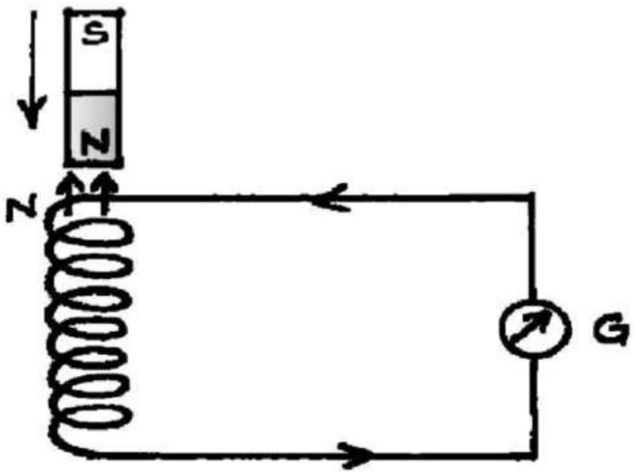
Lenz's Law & Energy Conservation

Lenz's Law is in accordance with the Law of Conservation of energy.

Explanation -

In the experimental verification of Lenz's Law, when N-pole of magnet is moved towards the coil, the upper face of the coil acquires north polarity. Therefore work has to be done against the force of repulsion, in bringing the magnet closer to the coil.

Similarly when north pole of magnet is moved away south polarity developes on the upper face of the coil. Therefore work has to be done against the force of attraction, in taking the magnet away from the coil.



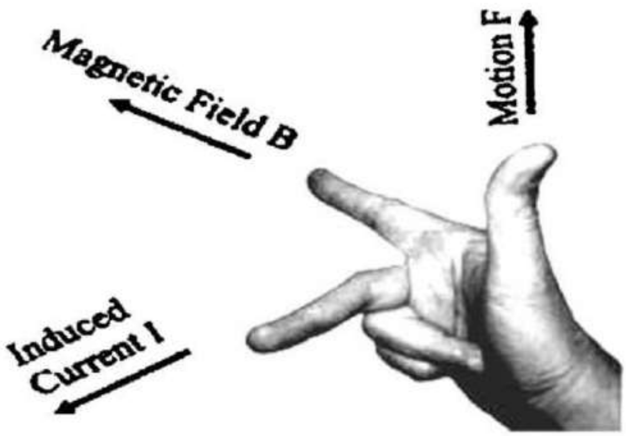
It is this mechanical work done in moving the magnet w.r.t. the coil that changes into electrical energy producing induced current. Thus energy is being transformed only.

When we do not move the magnet, work done is zero. Therefore, induced current is also not produced.

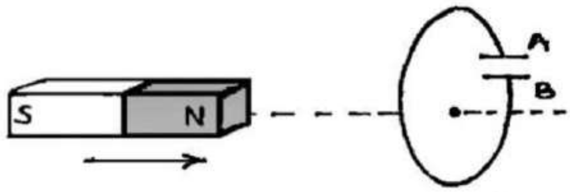
Fleming's Right Hand Rule

Fleming's right hand rule gives us the direction of induced e.m.f / current in a conductor moving in a magnetic field.

If we stretch the first finger, central finger and thumb of our right hand in mutually perpendicular directions such that first finger points along the direction of the field and thumb is along the direction of motion of the conductor, then the central finger would give us the direction of induced current.

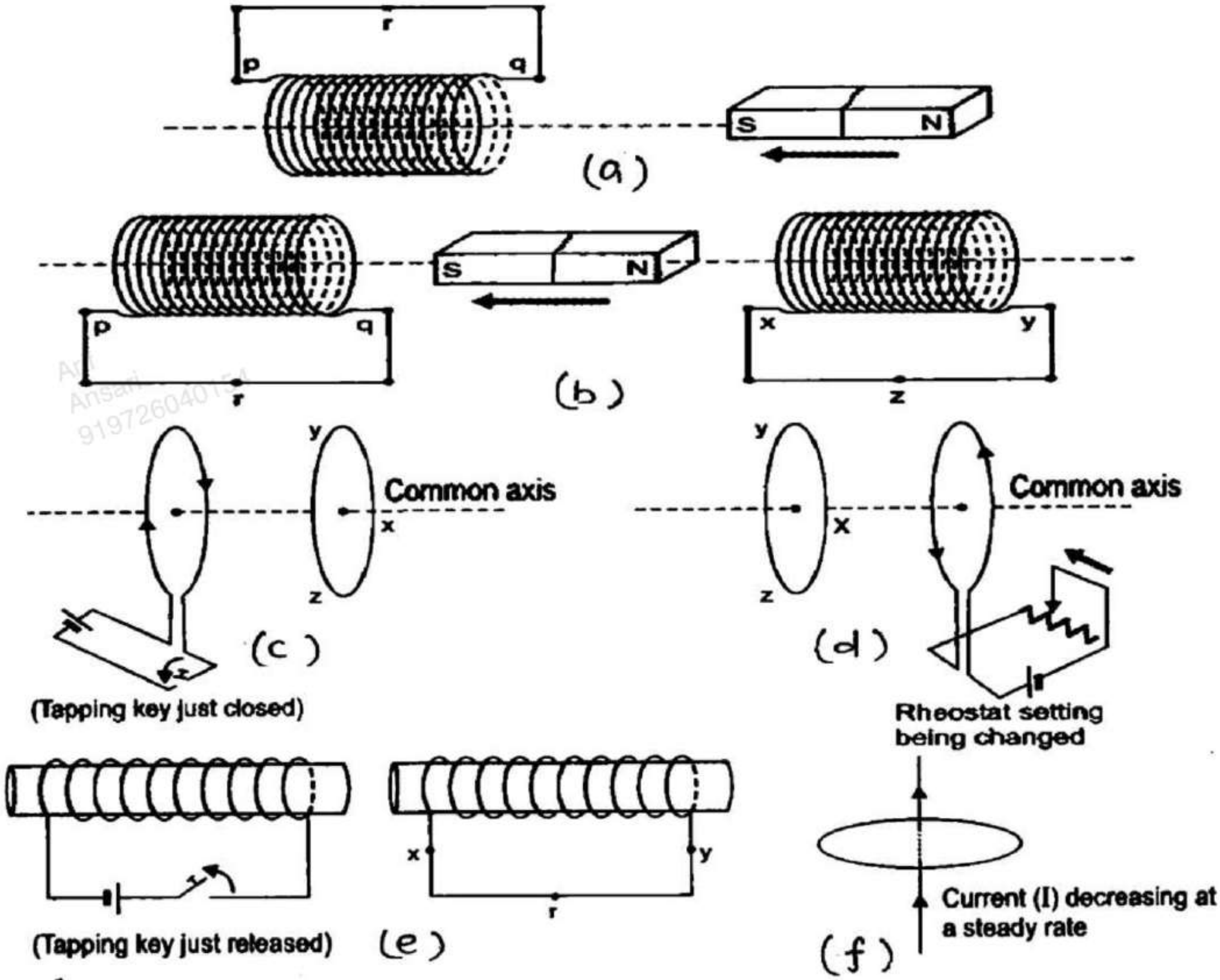


Q. (CBSE 2010) In the given figure a bar magnet is quickly moved towards a conducting loop having a capacitor. Predict the polarity of the plates A and B of the capacitor.



Sol. Here the north pole is approaching the magnet so the induced current in the face of loop viewed from left side will flow in such a way that it will behave like north pole so, south pole developes in loop when viewed from right hand side of the loop. The flow of induced current is clockwise, hence A acquires positive polarity and B negative.

Q. Predict the direction of induced current in the situations described by the following figures -



Sol.

(a) South pole develops at q, so current induced must be clockwise at q.

(b) Coil pq in this case would develop S-pole at q and coil XY would also develop S pole at X. So the induced current in coil pq will be from q to p and induced current in coil XY will be from X to Y.

(c) Induced current in the right loop will be along XYZ.

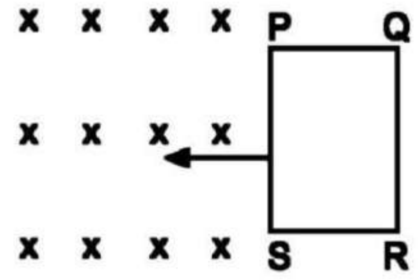
(d) Induced current in the left loop will be along ZYX as seen from front.

(e) Induced current in the right coil is from X to Y.

(f) No current is induced because magnetic lines of force lie in the plane of the loop.

Q. The closed loop PQRS is moving into a uniform magnetic field acting at right angles to the plane of the paper. State the direction of the induced current in the loop.

CBSE 2012 2019



Sol. Due to motion of coil the magnetic flux linked with the coil increases. So by Lenz's Law, the current induced in the coil will oppose this increase, hence produce a magnetic field upward, so current is induced in the coil will flow anticlockwise.

Q. A square loop of side 12 cm. with its sides parallel to X and Y axes is moved with a velocity of 8 cm/s in the positive x-direction in an environment containing a magnetic field in the positive z direction. The field is neither uniform in space nor constant in time. It has a gradient of 10^{-3} T/cm. along the negative x-direction and it is decreasing in time at the rate of 10^{-3} T/s. Determine the direction and magnitude of the induced current in the loop if its resistance is 4.5 mΩ.

Sol. Given that

$A = (12 \times 10^{-2})^2 = 144 \times 10^{-4}\ m^2$, Velocity $v = 8 \times 10^{-2}$ m/s

$\frac{dB}{dx} = 10^{-3}$ T/cm. , $\frac{dB}{dt} = 10^{-3}$ T/s , $R = 4.5 \times 10^{-3}\ \Omega$

Rate of change of magnetic flux due to variation in B with time-

$$e_1 = A\left(\frac{dB}{dt}\right) = 144 \times 10^{-4} \times 10^{-3} = 1.44 \times 10^{-5}\ \text{Wb/s}.$$

Rate of change of magnetic flux due to motion of the loop in a non-uniform magnetic field-

$$e_2 = A\left(\frac{dB}{dx}\right)\left(\frac{dx}{dt}\right) = 144 \times 10^{-4} \times 10^{-3} \times 8 = 11.52 \times 10^{-5}\ \text{Wb/s}$$

As both the effects cause a decrease in magnetic flux along the positive z-direction, therefore they add up.

$\therefore$ Total induced emf $(e) = e_1 + e_2$

$$e = 1.44 \times 10^{-5} + 11.52 \times 10^{-5}$$
$$e = 12.96 \times 10^{-5} \text{ V}$$

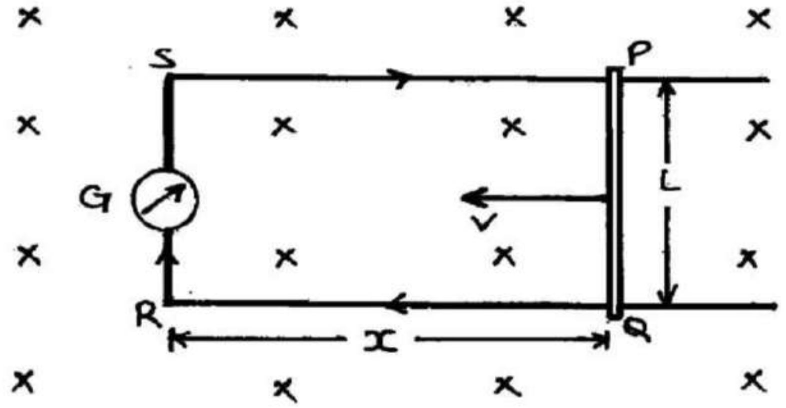
Induced Current $I = \frac{e}{R} = \frac{12.96 \times 10^{-5}}{4.5 \times 10^{-3}}$

$$I = 2.88 \times 10^{-2} \text{ A}$$

The direction of induced current is such as to increase the flux through the loop along positive z-direction, hence current will be seen to anticlockwise.

Motional Electromotive Force

Fig. Shows a rectangular conducting Loop PQRS in the plane of paper. The magnetic field is perpendicular to the plane of paper and directed inwards.



Let the conductor PQ moved towards the left with a velocity V, due to which the area enclosed by the loop PQRS decreases. Therefore amount of magnetic flux linked with the loop decreases and an e.m.f. is induced in the loop.

If at any time, the length $RQ = x$ and $PQ = L$, then magnetic flux linked with the loop PQRS is -

$$\phi = BLx$$

Then the induced e.m.f. in the loop can be given by -

$$e = \frac{-d\phi}{dt} = -\frac{d}{dt}(BLx)$$

$$e = BL\left(-\frac{dx}{dt}\right)$$

$$\boxed{e = BLV} \qquad \left(V = -\frac{dx}{dt}\right)$$

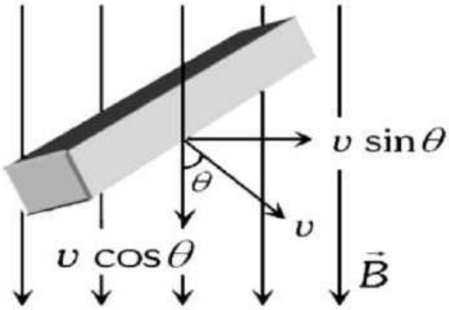
If R is resistance of loop at a given instant, the induced current I at that instant can be given as-

$$\boxed{I = \frac{e}{R} = \frac{BLV}{R}}$$

Arif Ansari 919726040154

* The direction of induced current is given by Fleming's right hand rule or by Lenz's Law.

Special Case - If rod is moving by making an angle θ with the direction of magnetic field



Induced emf

$$e = Bl(v\sin\theta)$$

Moving Conducting Rod in Earth's Magnetic Field -

1.| Horizontal Rod moving in horizontal Plane -

If its ends in
- E-W $\Rightarrow$ B_V Cuts
- N-S $\Rightarrow$ B_V Cuts

Motional EMF $\boxed{e = B_V l V}$

2.| Verticle rod moving in horizontal Plane -

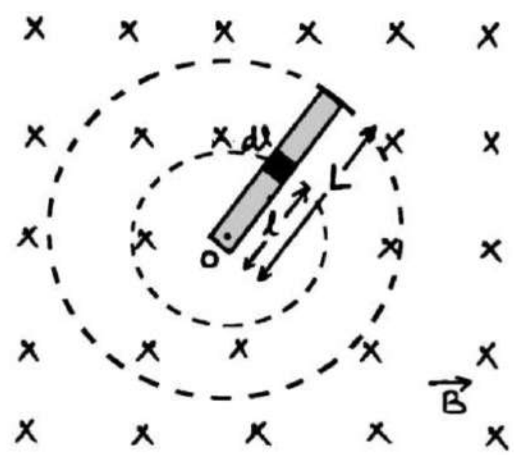
If it moves on
- E-W line $\Rightarrow$ B_H Cuts $\rightarrow$ $\boxed{e = B_H l V}$
- N-S line $\Rightarrow$ No Cutting $\rightarrow$ No Motional EMF

3.| Horizontal Rod allowed to fall under gravity -

If its end in
- E-W direction $\Rightarrow$ B_H Cuts $\rightarrow$ $\boxed{e = B_H l V}$
- N-S direction $\Rightarrow$ No cutting $\rightarrow$ No motional EMF

Induced EMF due to rotation of a conducting rod in uniform magnetic field -

Let a conducting rod of length L is rotating with angular velocity ω, perpendicular to a magnetic field $\vec{B}$.



To calculate the induced emf let consider an element of length dl at distance l from the end O of the rod.

If the linear velocity of the element dl is v the induced emf in it

$$de = Bv\,dl$$

Here element dl is rotating in a circular path of radius l, so that

$$v = l\omega$$

Hence $$de = B\omega l\,dl$$

To calculate the induced emf in the complete rod we integrate the above equation with limits 0 to L.

$$\int de = \int_0^L B\omega l\,dl$$

$$e = B\omega\left[\frac{l^2}{2}\right]_0^L$$

$$e = B\omega\left[\frac{L^2}{2} - 0\right]$$

or $$\boxed{e = \frac{B\omega L^2}{2}}$$

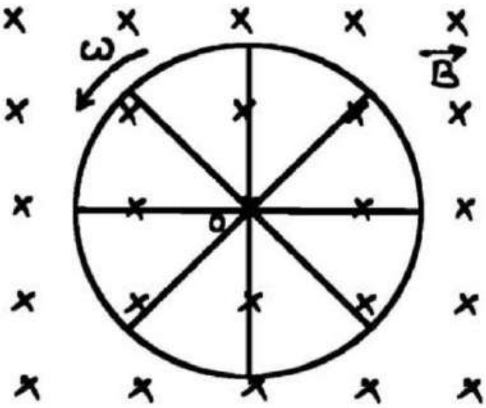
As $\omega = 2\pi f$ so that $e = \frac{1}{2}B \times 2\pi f \times L^2$

or $$\boxed{e = B\pi L^2 f = BAf}$$

where $A = \pi L^2$ = Area of the coil

Note - If a conducting rod rotating about an axis perpendicular to its length and passing through its centre then the points which are at equal distance from the centre have the same potential and their potential difference will be zero.

Induced emf in a wheel of cycle rotating in a uniform magnetic field -



If a wheel of cycle having spokes of length l is rotating with angular velocity ω, perpendicular to a magnetic field $\vec{B}$, then due to induced emf in each spoke behaves like a cell of emf e.

As these cells are connected in parallel then

$$e_{equivalent} = e \text{ (emf of each cell)}$$

$$\boxed{e_{equivalent} = \frac{1}{2} B\omega l^2}$$

* Net induced emf does not depends on the number of spokes in the wheel.

[redacted]

Q. A metallic rod of 1 m length is rotated with a frequency of 50 rev/s, with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius 1 m, about an axis passing through the centre and perpendicular to the plane of the ring. A constant uniform magnetic field of 1 T parallel to the axis is present everywhere. What is the e.m.f. between the centre and the metallic ring?

Sol. Here, radius of circle $R = 1\,m$.

Frequency $\nu = 50$ rev/s , $B = 1\,T$

$$e = \frac{1}{2} B R^2 \omega$$

$$e = \frac{1}{2} B R^2 (2\pi\nu)$$

$$= \pi \nu B R^2$$

$$e = 3.14 \times 50 \times 1 \times (1)^2$$

$$= 157.1\ V$$

*2007 **Q.** A Jet plane is travelling west at the speed of 1800 Km/h. What is the voltage difference developed between the ends of the wing 25 m long if the earth's magnetic field at the location has a magnitude of $5 \times 10^{-4}\,T$ and the dip angle is $30°$?

Sol. Given that - $V = 1800$ Km/h $= 500$ m/s.

$L = 25\,m$, $B = 5 \times 10^{-4}\,T$, $\delta = 30°$

So $e = B_V L V$ ($B_V \rightarrow$ Verticle Component of earth's field)

$$e = (B \sin\delta) L V$$

$$= 5 \times 10^{-4} \times \sin 30° \times 25 \times 500$$

$$e = 3.1\ \text{Volt}$$

Q. A circular copper disc of radius 10 cm, rotates at a speed of 20π rad/s about an axis through its centre and perpendicular to the disc. A uniform magnetic field of 0.2 T acts perpendicular to the disc.

(i) Calculate potential difference developed between the axis of the disc and the rim.

(ii) What is the induced current if the resistance of disc is $2\ \Omega$?

Sol. Here $r = 10\text{ cm} = 10^{-1}\text{ m}$. $\omega = 20\pi$ rad/s

$B = 0.2\text{ T}$, $R = 2\ \Omega$

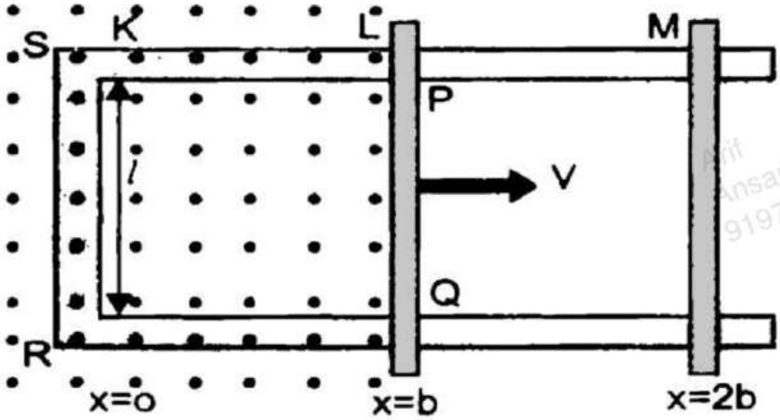
So $e = \frac{1}{2} B \omega r^2$

$$e = \frac{1}{2} \times 0.2 \times 20\pi \left(10^{-1}\right)^2 = 0.0629\text{ V}$$

$$i = \frac{e}{R} = \frac{0.0629}{2} = 0.0315\text{ A}$$

Q. The arm PQ of the rectangular conductor is moved from $x = 0$ to the right side as shown in figure. The uniform magnetic field is perpendicular to the plane and extends from $x = 0$ to $x = b$ and is zero for $x > b$. Only the arm PQ possesses substantial resistance r. Consider the situation when the arm PQ is pulled outwards from $x = 0$ to $x = 2b$, and is then moved back to $x = 0$ with constant speed v. Obtain expression for the flux, the induced e.m.f., the necessary force to pull the arm and the power dissipated as Joule heat. Sketch the variation of these quantities with distance.

Sol.



Consider the forward motion from $x=0$ to $x=2b$. When arm PQ is at x, magnetic flux linked with the circuit PQRS is -

$$\phi = Blx \quad \text{for } 0 \le x < b$$

$$\phi = Blb \quad \text{for } b \le x \le 2b$$

So the induced e.m.f. is -

$$e = \frac{-d\phi}{dt} = -Blv \quad \text{for } 0 \le x < b$$

$$e = -\frac{d\phi}{dt} = 0 \quad \text{for } b \le x \le 2b$$

When the induced e.m.f is non-zero, the magnitude of current is $I = \frac{e}{r} = \frac{Blv}{r}$

Force required to keep the arm PQ in constant motion $= BIl$

$$F = B\left(\frac{Blv}{r}\right)l$$

$$= \frac{B^2l^2v}{r}, \text{ for } 0 \le x < b$$

$$F = \text{zero}, \text{ for } b \le x \le 2b$$

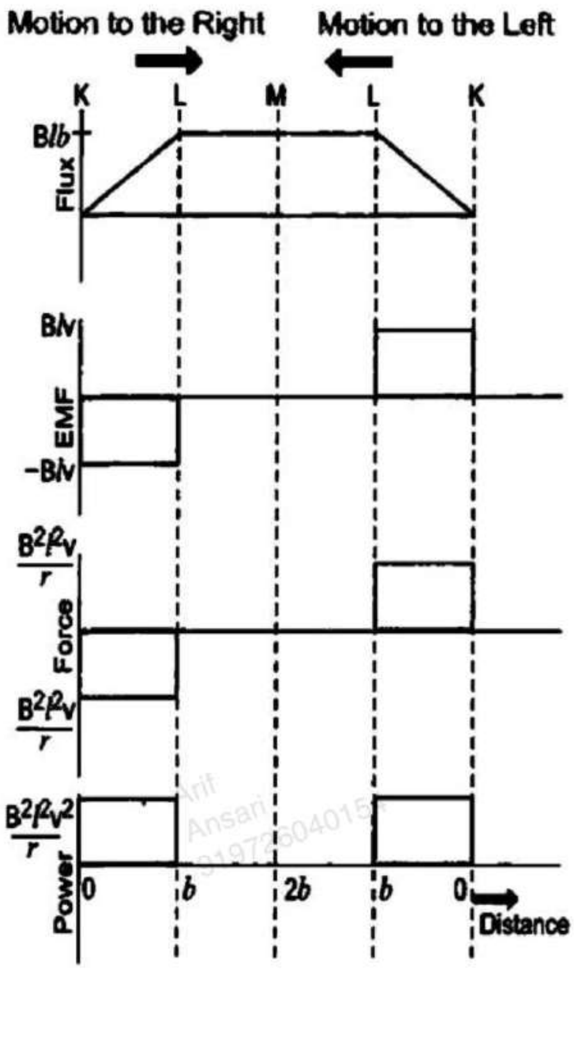
The direction of the force on PQ is to the left

The Joule heating loss is -

$$P = I^2r = \frac{B^2l^2v^2}{r}$$

$$P = \frac{B^2l^2v^2}{r}, \quad \text{for } 0 \le x < b$$

$$P = 0, \quad \text{for } b \le x \le 2b$$



Similar expressions are obtained for inward motion of PQ from $x=2b$ to $x=0$.

Energy Consideration in Motional E.M.F.

When a conductor of length l is moved with a velocity v in a perpendicular magnetic field B, then the motional e.m.f. produced in the conductor is

$$e = Blv$$

Let r be the resistance of movable arm PQ of the rectangular conductor. Here we assume that the resistance of remaining arm QR, RS and SP is negligible compared to r.

$$\boxed{\text{Current Induced in the loop } I = \frac{e}{r} = \frac{Blv}{r}}$$

The magnitude of force on the conductor PQ moving in the magnetic field is -

$$F = BIl = B\left(\frac{Blv}{r}\right)l = \frac{B^2l^2v}{r}$$

The direction of this force is opposite to the velocity of the conductor.

Power required to push the conductor is

$$P_M = F \times v = \frac{B^2l^2v}{r} \times v = \frac{B^2l^2v^2}{r}$$

As the conductor is pushed mechanically, the mechanical energy dissipated per second is given by

$$P_E = I^2r = \left(\frac{Blv}{r}\right)^2 = \frac{B^2l^2v^2}{r}$$

which is the same as the power required to push the conductor.

Hence, mechanical energy required to move the conductor PQ is converted into electrical energy first (i.e. the induced e.m.f.) and then to thermal energy.

$$\boxed{\text{Heat energy produced/Sec.} = \frac{B^2l^2v^2}{r}}$$

Q. A coil of number of turns N, area A is rotated at a constant angular speed ω, in a uniform magnetic field B and connected to a resistor R. Deduce the expression for -

Delhi 2008

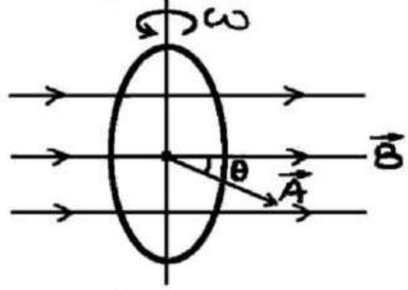
(a) Maximum emf induced in the coil
(b) Power dissipation in the coil.

Sol. Let initially area vector $\vec{A}$ of coil makes an angle θ with the direction of magnetic field.

Let the coil rotates by an angle θ with the magnetic field in time t.

$$\therefore \quad \omega = \frac{\theta}{t}$$

$$\text{or} \quad \theta = \omega t$$



$\therefore$ Magnetic flux linked with each turn of the rectangular coil is -

$$\phi = \vec{B}\cdot\vec{A} = BA\cos\theta = BA\cos\omega t$$

$$\therefore \quad \frac{d\phi}{dt} = -BA\omega\sin\omega t \qquad [\omega = 2\pi f]$$

For N turns of the coil by Faraday's Law -

$$e = -N\frac{d\phi}{dt} = NBA\omega\sin\omega t$$

$$\text{or} \quad e = e_0 \sin\omega t$$

where e_0 is maximum or peak value of emf induced in coil and given by -

$$\boxed{e_0 = NBA\omega = NBA(2\pi f)}$$

(b) Power dissipated $(P) = \frac{V_{rms}^2}{R} = \frac{e_{rms}^2}{R}$

where R is the resistance.

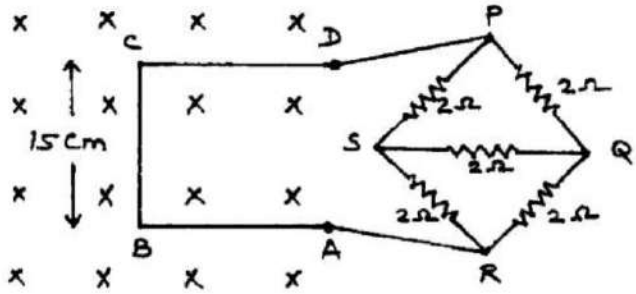
Arif Ansari 919726040154

$$\because \quad e_{rms} = \frac{e_0}{\sqrt{2}} = \frac{NBA\omega}{\sqrt{2}}$$

$$\therefore \quad P = \frac{(NBA\omega/\sqrt{2})^2}{R} \quad \text{or} \quad \boxed{P = \frac{N^2B^2A^2\omega^2}{2R}}$$

Q. CBSE 2006

A Metallic square loop ABCD of size 15 cm. and resistance 1 Ω is moved at a uniform velocity of v m/s in a uniform magnetic field of 2T, the field lines being normal to the plane of the paper. The loop is Connected to an electrical network of resistors, each of resistance 2 Ω. Calculate the speed of the loop for which 2mA current flows in the loop.



Sol: Given - $l = 15\,cm$, $B = 2T$, $R_{ABCD} = 1\,\Omega$, $I = 2mA$

∵ PQRS is a balanced wheatstone bridge.

∴ SQ wire of 2 Ω resistance can be removed.

∴ The equivalent resistance across PR is given by-

$$R = \frac{4 \times 4}{4+4} = 2\,\Omega$$

Net resistance of the circuit $= 1 + 2 = 3\,\Omega$

$$\therefore \quad V = IR = 2 \times 10^{-3} \times 3 = 6 \times 10^{-3} \text{ Volt.}$$

But potential difference across conductor $= vBl$

So $$vBl = 6 \times 10^{-3}$$

$$v = \frac{6 \times 10^{-3}}{2 \times 0.15} = 2 \times 10^{-2} \text{ m/s.}$$

Eddy Currents

The eddy currents are basically the currents induced in the body of a conductor due to change in magnetic flux linked with the conductor.

For example when we move a metal plate out of a magnetic field, then the relative motion of the field and the conductor induces a current in the conductor.

The conduction electrons making up the induced current whirl about within the plate as if they were caught in an eddy (or whirlpool) of water. This is called the eddy current.

$$\text{Eddy Current } (i) = \frac{\text{induced e.m.f } (e)}{\text{resistance } (R)}$$

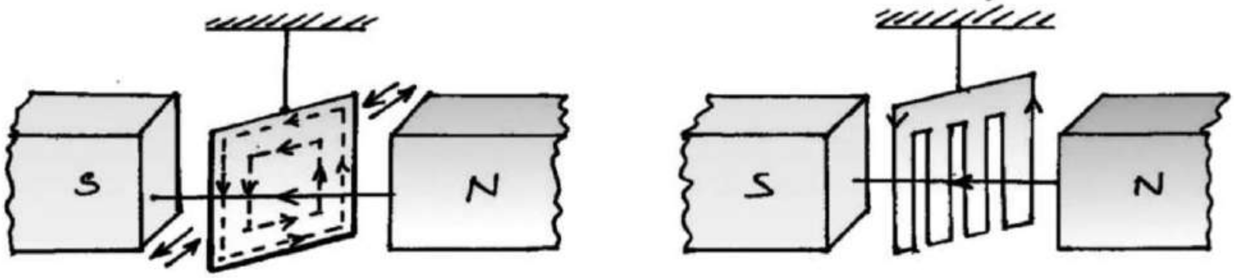
But we know that $e = -\frac{d\phi}{dt}$ so

$$\boxed{i = -\frac{d\phi/dt}{R}}$$

The direction of eddy current is given by Lenz's law or Fleming's right hand rule.

Experimental Demonstration

Suspend a flat metallic plate between pole pieces N and S of an electromagnet. Let it come to rest with its plane perpendicular to magnetic field.



When the magnetic field is off, the metallic plate disturbed from it's equilibrium position and left, oscillates freely for a longer time.

But when the electromagnet is switched on, the vibrations of the plate are damped and the plate stops vibrating sooner. This is because of eddy currents developed in the vibrating plate

Reason :-

In the normal position of rest of the plate, magnetic flux linked with the plate is maximum. When it is displaced towards any one extreme position, are of plate in the field decreases. Therefore magnetic flux linked with the plate decreases. Eddy currents develop in the plate which, according to Lenz's law, oppose the decrease in flux and hence the motion of the plate towards extreme position.

Similarly when plate returns from extreme position to mean position, area of plate in the field increases, magnetic flux linked with the plate increases. Eddy currents are developed which oppose the increase in flux and hence the motion of the plate towards the mean position.

In either case, vibrations of the plate are opposed and hence damped.

How to reduce eddy currents -

When a plate with slots cut in it is made to oscillate in the magnetic field, the damping effect is there, but it is much smaller compared to the case when no slots were cut.

This means eddy currents are reduced. This is because magnetic moment of the induced current (which opposes the motion) depend upon the area enclosed by the currents.

$$\boxed{\vec{M} = I\vec{A}}$$

Hence to minimize the eddy currents, the metal core to be used in an appliance like dynamo, transformer, choke coil, motor etc. is taken in the form of thin steets. Each sheet is electrically insulated from the other by insulating varnish. Such a core is called a Laminated core.

Large resistance between the thin sheets confines the eddy currents to the individual sheets. Hence the eddy currents are reduced to a large extent.

Uses of Eddy Currents

(1.) Induction Furnace -

The heating effect of eddy currents is used for melting a metal in an induction furnace. Eddy currents of large magnitude are produced by varying magnetic field. The changes in the magnetic field are so rapid that very large eddy currents are generated and heat produced is sufficient to melt a metal quickly.

(2.) Electric Brakes -

It is an efficient system of brakes employed in electric trains. The axle of a train is surrounded by a coaxial cylinderical drum. When the train is to be stopped, a strong magnetic field is applied to the rotating drum. This results in the generation of the large eddy currents which oppose the relative motion and thus the train slows down and comes to rest quickly.

(3.) Speedometer -

In a speedometer, the magnet rotates inside a pivoted metallic drum. The speed of rotation depends depends on the speed of the vehicle. Eddy current in the drum causes the magnet to rotate with the drum. It rotates through an angle which is proportional to the speed of the vehicle. A pointer attached to the magnet gives a reading of the speed of the vehicle on a calibrated scale.

(4.) Deadbeat Galvenometer -

When the current flows in the coil of a moving coil galvenometer, the coil gets deflected and as soon as the current stops the coil tends to oscillate about its mean position. However the eddy currents generated in the metal core of the coil oppose the oscillatory motion, due to which the coil comes to rest quickly.

Q. Mention any two useful applications of eddy currents.

Delhi 2009

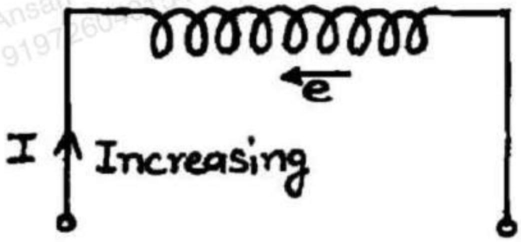
Sol. (a) Magnetic brake

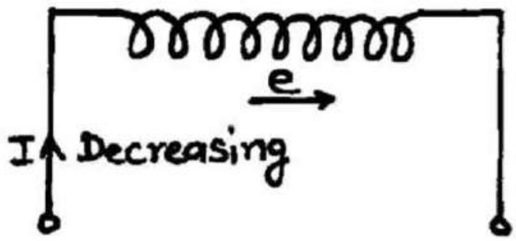
(b) Magnetic Furnace / Induction cooker.

Self Induction

It is the property of a coil by virtue of which the coil opposes any change in the strength of current flowing through it by inducing an e.m.f. in itself.

When current (I) is increasing, the self induced e.m.f. (e) appears accross the coil in a direction such that it opposes the increase. Therefore it would be in a direction opposite to I.





when current (I) is decreasing, the self induced e.m.f. (e) appears across the coil in a direction, such that it opposes the decrease. Therefore it would be in the direction of I.

Self Inductance -

Let the current I flowing through a coil at any time and ϕ is the amount of magnetic flux linked with all the turns of the coil at that time. Then it is found that

$$\phi \propto I \quad \text{or} \quad \boxed{\phi = LI}$$

Here L is a constant of proportionality and is called self inductance or coefficient of self induction of the coil.

> * The value of L depends upon the number of turns, area of cross section and nature of material of the core on which the coil is wound.

If $I = 1$, $\phi = L \times 1$ or $\boxed{L = \phi}$

Self inductance of a coil is numerically equal to the amount of magnetic flux linked with the coil when unit current flows through the coil.

The e.m.f. induced in the coil is given by.

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(LI)$$

or

$$\boxed{e = -L \times \frac{dI}{dt}}$$

* The SI unit of L is henry.

$$L = \frac{-e}{dI/dt} \qquad \therefore \quad 1 \text{ henry} = \frac{1 \text{ Volt}}{1 \text{ ampere / sec.}}$$

The self inductance of a coil is said to be 1 henry when a current change at the rate of 1 ampere/sec through the coil induces an e.m.f. of 1 volt in the coil.

Q. (CBSE 2011) Current in a circuit falls steadily from 2 A to 0 A in 10 ms. If an average emf of 200 V is induced, calculate the self inductance of the circuit.

Sol. Given that -

$\Delta I = -2\text{ A}$, $\Delta t = 10 \times 10^{-3}\text{ s}$, $V = 200\text{ V}$

$\because$ Induced emf $e = -L\frac{\Delta I}{\Delta t}$

$$L = -e\frac{\Delta t}{\Delta I} = -\frac{200 \times 10 \times 10^{-3}}{-2} = 1\text{ H}$$

$\therefore$ Self Inductance $L = 1\text{ H}$

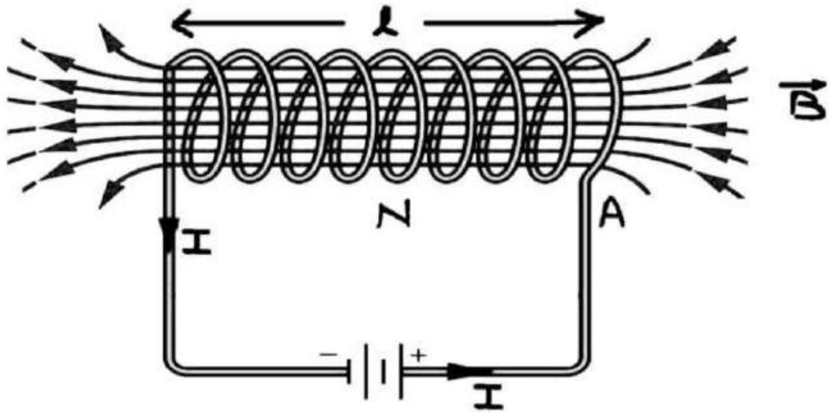
Self Inductance of a Solenoid (CBSE 2009, 12, 19)

Consider a long air solenoid having number of turns per unit length n and current flowing in it is I.

where $n = \frac{N}{l}$

Magnetic field within the solenoid $B = \mu_0 n I$

Where $\mu_0 = 4\pi \times 10^{-7}$ H/m



If A is cross-sectional area of solenoid, then flux linked with solenoid of length l is

$$\phi = NBA\cos 0^\circ$$

$$\phi = N \times \mu_0 n I A = N \times \mu_0 \frac{N}{l} I A$$

$$\phi = \frac{\mu_0 N^2 A I}{l}$$

Self inductance of air solenoid

$$L = \frac{\phi}{I}$$

$$\boxed{L = \frac{\mu_0 N^2 A}{l}}$$

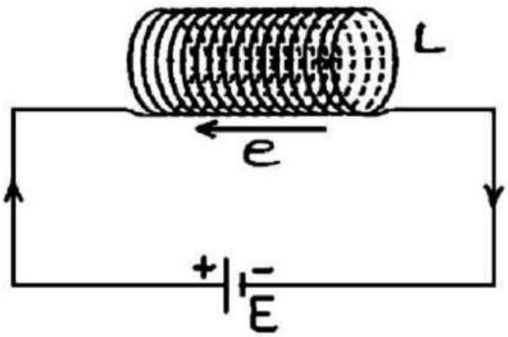
If solenoid contains a core of ferromagnetic substance of relative permeability μ_r, then self inductance

$$\boxed{L = \frac{\mu_r \mu_0 N^2 A}{l}}$$

Q. A source of emf E is used to establish a current I, through a coil of self inductance L. Show that the work done by the source of build up the current I is $\frac{1}{2}LI^2$.

Delhi 2010

Sol. The source of emf E establishes current in coil in opposition of induced emf (Lenz's law) and hence does work for the same. This work done by sources stores in the form of magnetic energy in the coil.



Let I current flows through the coil of self inductance L at any instant t when rate of change of current in coil is dI/dt.

$\therefore$ Induced emf $e = -L\frac{dI}{dt}$

Work done in establishing the current in small time interval dt is given by-

$$dw = P \cdot dt = -EI\,dt$$

$$dw = -\left(-L\frac{dI}{dt}\right) I\,dt = LI\,dI$$

$\therefore$ Total work done in increasing the current from zero to I is -

$$W = \int_0^I LI\,dI = L\int_0^I I\,dI$$

$$W = L\left[\frac{I^2}{2}\right]_0^I$$

$$W = \frac{1}{2}L\left[I^2 - 0^2\right]$$

$$\boxed{W = \frac{1}{2}LI^2}$$

* After the current has reached its final steady state value I, $\frac{dI}{dt} = 0$ and no more energy is input to the inductor.

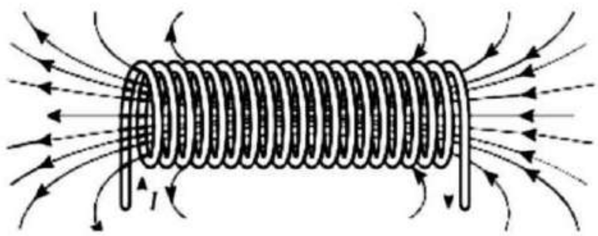
* When the current decreases from I to zero, the inductor acts as a source that supplies a total amount of energy $\frac{1}{2}LI^2$ to the external circuit

If we interrupt the circuit suddenly by opening a switch the current decreases very rapidly, the induced emf is very large-and the energy may be dissipated in an arc in the switch.

Magnetic Energy density –

Energy stored per unit volume is called energy density of the inductor.

$$u = \frac{U}{V} = \frac{\frac{1}{2}LI^2}{V}$$

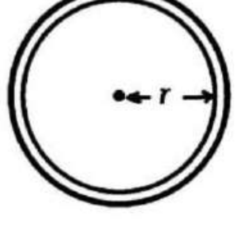
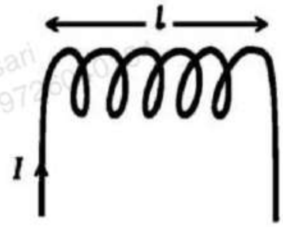


$$= \frac{\frac{1}{2}\left(\frac{\mu_0 N^2 A}{l}\right) \times I^2}{A \times l}$$

$$u = \frac{1}{2}\mu_0 \frac{N^2}{l^2} I^2 = \frac{1}{2}\mu_0 n^2 I^2$$

$$\Rightarrow \boxed{u = \frac{1}{2}\frac{B^2}{\mu_0}}$$

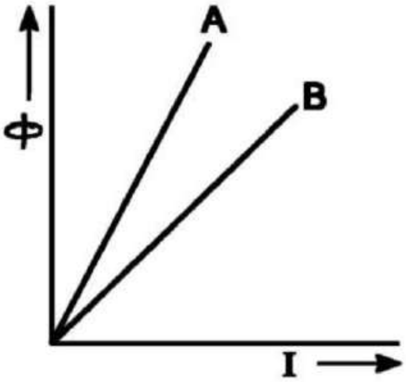
various formulae for *L*

Circular coil $L = \frac{\mu_0 \pi N^2 r}{2}$ 	Solenoid $L = \frac{\mu_0 \mu_r N^2 A}{l} = \frac{\mu N^2 A}{l}$ $(\mu = \mu_0 \mu_r)$ 

Q. (CBSE 2013) A plot of magnetic flux (ϕ) versus current (I) is shown in the fig. for two inductors A and B. Which of the two has large value of self-inductance.

Sol. We know that $\phi = LI$

or $\frac{\phi}{I} = L$



The slope of $\frac{\phi}{I}$ of straight line is equal to self inductance L. It is larger for inductor A, therefore inductor A has larger value of self inductance L.

Q. (CBSE 2014) The current flowing in the two coils of self-inductance $L_1 = 16\,mH$ and $L_2 = 12\,mH$ are increasing at the same rate. If the power supplied to the two coils are equal, Find the ratio of (i) induced voltage (ii) the current (iii) the energies stored in the two coils at a given instant.

Sol. (i) Induced emf $e = -L\frac{dI}{dt}$

$$\therefore \quad \frac{e_1}{e_2} = \frac{-L_1\frac{dI}{dt}}{-L_2\frac{dI}{dt}} = \frac{L_1}{L_2}$$

$$\frac{e_1}{e_2} = \frac{16\,mH}{12\,mH} = \frac{4}{3}$$

(ii) Power supplied $P = e \times I$

Since power is same for both the coils

$$\therefore \quad e_1 I_1 = e_2 I_2 \Rightarrow \frac{I_1}{I_2} = \frac{e_2}{e_1} = \frac{3}{4}$$

(iii) Energy stored in the coil $U = \frac{1}{2}LI^2$

$$\therefore \quad \frac{U_1}{U_2} = \frac{\frac{1}{2}L_1 I_1^2}{\frac{1}{2}L_2 I_2^2}$$

$$\frac{U_1}{U_2} = \frac{L_1}{L_2} \times \frac{I_1^2}{I_2^2} = \frac{4}{3} \times \left(\frac{3}{4}\right)^2 = \frac{3}{4}$$

Q. A 0.5 m long solenoid of 10 turns/cm has area of cross-section $1\,cm^2$. Calculate the voltage induced across its ends if the current in the solenoid is changed from 1A to 2A in 0.1 sec.

CBSE 2019

Sol: Given that $I_1 = 1A$, $I_2 = 2A$, $l = 0.5m$

$$n = \frac{N}{l} = 10 \text{ turns/cm} = 1000 \text{ turns/m}$$

$$A = 1\,cm^2 = 10^{-4}\,m^2, \quad dt = 0.1 \text{ sec.}$$

The induced voltage $|e| = L\frac{dI}{dt}$

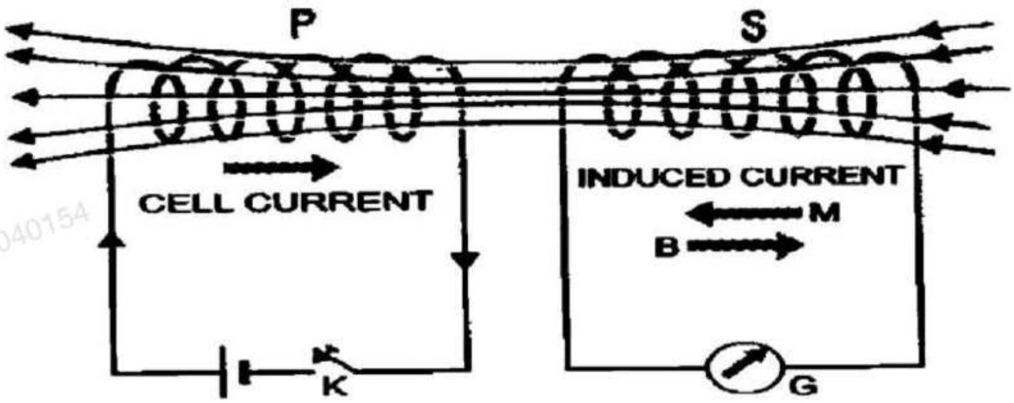
$$= \frac{\mu_0 N^2 A}{l} \times \frac{dI}{dt} = \mu_0 n^2 A l \times \frac{dI}{dt}$$

$$e = 4\pi \times 10^{-7} \times (1000)^2 \times 10^{-4} \times 0.5 \times \frac{(2-1)}{0.1}$$

$$= 20\pi \times 10^{-5}\ mV$$

Mutual Inductance

It is the property of two coils by virtue of which each opposes any change in the strength of current flowing through the other by developing an induced e.m.f.



On pressing or releasing K, galvanometer shows a sudden temporary deflection. This is due to mutual induction as detailed below –

(i) On pressing Key K, current in P increases from zero to maximum value. It takes some time. During this time, current in P is increasing. Therefore magnetic flux linked with P is increasing.

As S is closeby, magnetic flux associated with S also increases. An e.m.f. is induced in S. According to Lenz's Law, the induced current in S would oppose increase in current in P by flowing in a direction opposite to the cell current in P.

(ii) On releasing Key K, current in P decreases from maximum to zero value. It takes some time. During this time current in P is decreasing. Therefore magnetic flux linked with P is decreasing. As S is held closeby, magnetic flux associated with S also decreases. An e.m.f is induced in S.

According to lenz's Law induced current in S during break flows in the direction of the cell current in P so as to oppose the decrease n current in P.

Mutual Inductance

Let I is the strength of current in one coil and ϕ is the total amount of magnetic flux linked with all the turns of the neighbouring coil.
Then it is found that -

$$\phi_2 \propto I_1 \qquad \text{or} \qquad \boxed{\phi_2 = M I_1}$$

where M is a constant of proportionality and is called coefficient of mutual induction or mutual inductance of the two coils.

If $I_1 = 1$, then $\boxed{\phi_2 = M}$

Thus the mutual inductance of two coils is equal to the amount of magnetic flux linked with one coil when unit current flows through the neighbouring coil.

The induced e.m.f. in the neighbouring coil is given by -

$$e_2 = -\frac{d\phi_2}{dt} = -\frac{d}{dt}(MI_1)$$

$$\boxed{e_2 = -M\frac{dI_1}{dt}}$$

SI Unit of M is henry

If $dI_1/dt = 1$ then $e_2 = -M$

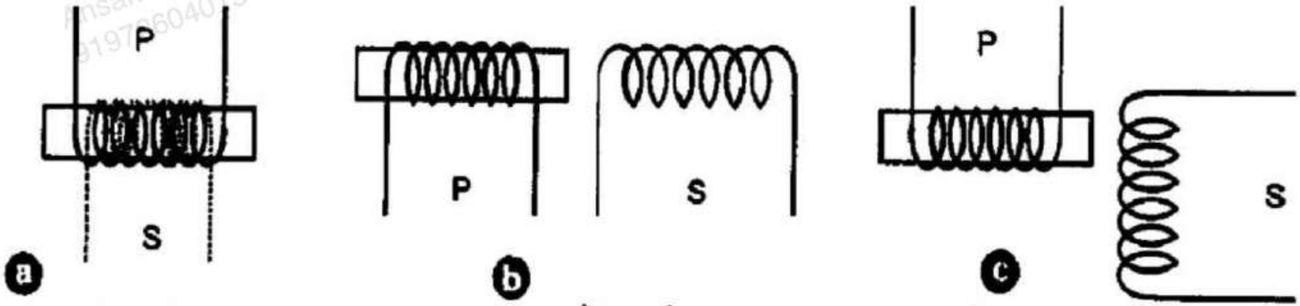
So the mutual inductance of two coils is said to be one Henry, when a current changes at the rate of one ampere/sec. in one coil induces an e.m.f. of one volt in the other coil.

Coefficient of Coupling (K)

Coefficient of coupling (K) of two coils is a measure of the coupling between the two coils. It is given by -

$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$

where L_1 and L_2 are coefficients of self inductance of the two coils and M is coefficient of mutual inductance of the two coils.



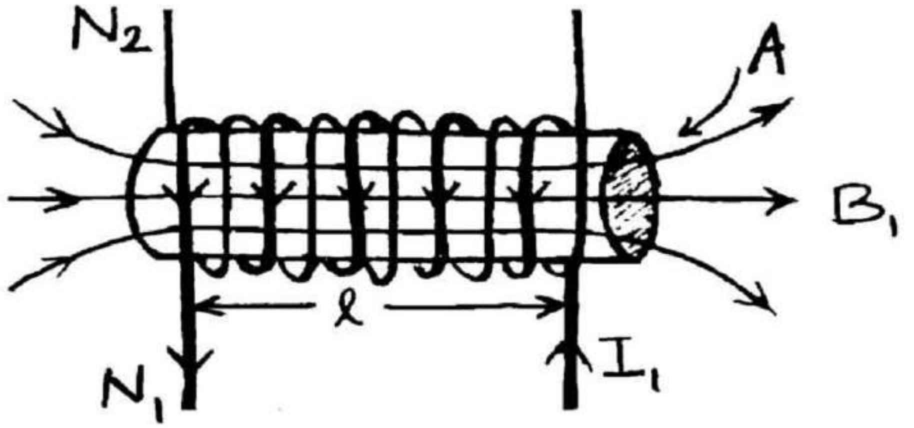
K=1 K<1 K=0

- ★ The value of K ranges from 0 to 1 $(0 \le K \le 1)$
- ★ Value of K decided from fashion of coupling.
- ★ K is also defined as

$$K = \frac{\phi_s}{\phi_p} = \frac{\text{magnetic flux linked with secondary Coil}}{\text{magnetic flux linked with primary Coil}}$$

Mutual Inductance of two coaxial Solenoids

Consider two long co-axial solenoids, each of length l. Let N_1 and N_2 are the number of turns in the solenoids. and cross sectional area of the coils is A.



Let the current in the first solenoid is I_1 then the magnetic field produced by it is

$$B_1 = \mu_0 n_1 I_1 = \mu_0 \frac{N_1}{l} I_1$$

So that the magnetic flux passing through the second coil is

$$\phi_2 = N_2 B_1 A \cos 0^\circ$$

or

$$\phi_2 = N_2 \times \mu_0 \frac{N_1}{l} I_1 A$$

Mutual inductance of the two solenoids is

$$M = \frac{\phi_2}{I_1}$$

$$\boxed{M = \frac{\mu_0 N_1 N_2 A}{l}}$$

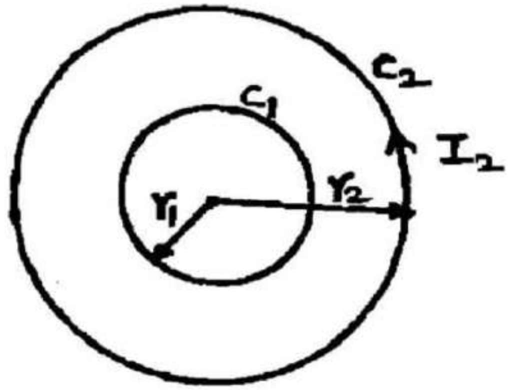
If solenoid contains a core of ferromagnetic material of relative magnetic permeability μ_r then

$$\boxed{M = \frac{\mu_r \mu_0 N_1 N_2 A}{l}}$$

Q. Two concentric circular coils c_1 and c_2 of radius r_1 and r_2 ($r_1 << r_2$) respectively are kept coaxially. If current is passed through c_2, then find an expression for mutual inductance between the coils.

CBSE 2009 2011

Sol. In case of two concentric coils when current flows through one coil, the emf is induced in the other coil.



Let I_2 current passes through the coil c_2.
Magnetic field at centre due to current loop c_2

$$B_2 = \frac{\mu_0 N_2 I_2}{2r_2}$$ [N_2 = number of turns in coil c_2.]

$\therefore$ Total magnetic flux linked with coil c_1

$$\phi_1 = N_1 B_2 A_1$$

$$\phi_1 = N_1 \left(\frac{\mu_0 N_2 I_2}{2r_2}\right)(\pi r_1^2)$$

But $$\phi = M I_2$$

so that $$M I_2 = \frac{\mu_0 \pi N_1 N_2 r_1^2 I_2}{2r_2}$$

or $$\boxed{M = \frac{\mu_0 \pi N_1 N_2 r_1^2}{2r_2}}$$

Combination of Inductance

(1) **Series :** If two coils of self-inductances L_1 and L_2 having mutual inductance are in series and are far from each other, so that the mutual induction between them is negligible, then net self inductance

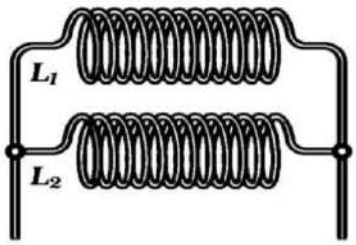
$$L_S = L_1 + L_2$$

When they are situated close to each other, then net inductance $L_S = L_1 + L_2 \pm 2M$

(2) **Parallel :** If two coils of self-inductances L_1 and L_2 having mutual inductance are connected in parallel and are far from each other, then net inductance L is

$$\frac{1}{L_P} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$\Rightarrow \quad L_P = \frac{L_1 L_2}{L_1 + L_2}$$



When they are situated close to each other, then

$$L_P = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M}$$

Q.1. A square plate of side length 4cm is placed in a magnetic field of induction 20 mT in a direction making an angle 30^0 with the plane of coil then the magnetic flux linked with the coil is?

Solution:

$$\text{Area} = \left(4\times10^{-2}\right)\times\left(4\times10^{-2}\right) = 16\times10^{-4}\,m^2$$

$$\theta = 90 - 30 = 60^0 \qquad \vec{B} = 20\times10^{-3}\,T$$

$$\phi = BA\cos\theta$$

$$= \left(20\times10^{-3}\right)\left(16\times10^{-4}\right)\times\frac{1}{2}$$

$$= 10\times10^{-3}\times16\times10^{-4} = 16\times10^{-6} = 16\ \mu\ \text{wb}$$

Q.2. A circular loop of area $20\,cm^2$ is placed on x-y plane. Containing uniform magnetic field of Induction $\vec{B} = 0.4\hat{i} + 0.3\hat{j}$ Tesla then the magnetic flux linked with the coil is?

Solution: $\vec{B} = 0.4\hat{i} + 0.3\hat{j}\ T\ ;\ \vec{A} = 20\times10^{-4}.\hat{k}\,m^2$

$$\phi = \vec{B}.\vec{A} = \left(0.4\hat{i} + 0.3\hat{j}\right).\left(20\times10^{-4}\hat{k}\right)$$

$$= \left(0.4\hat{i} + 0.3\hat{j}\right).\left(0\hat{i} + 0\hat{j} + 20\times10^{-4}\hat{k}\right)$$

$$= 0 + 0 + 0 = 0$$

$\therefore$ flux linked with the coil is zero

Q.3. A wire loop of 5 turns area vector $\vec{A} = \left(4\hat{i} + 3\hat{j}\right)m^2$ is placed in a magnetic field of Induction $\vec{B} = 0.4\hat{j}\ T$. Then the flux linked with the coil is?

Solution:

$$\vec{B} = 0.4\hat{j}\ ;\ \vec{A} = 4\hat{i} + 3\hat{j}\ ;\ \text{and } N = 5$$

$$\phi = N\left(\vec{B}.\vec{A}\right)$$

$$= 5\left(0\hat{i} + 0.4\hat{j}\right).\left(4\hat{i} + 3\hat{j}\right) = 5(0 + 1.2) = 6\ wb$$

Q.4. ***A rectangular loop of area 0.06 m² is placed in a uniform magnetic field of 0.3T with its plane (i) normal to the field (ii) inclined 30⁰ to the field (iii) parallel to the field. Find the flux linked with the coil in each case.***

Solution: $\phi = NBA\cos\theta$

i) $\phi = 1 \times 0.06 \times 0.3 \times \cos 0^0 = 0.018$ wb

ii) $\phi = 1 \times 0.06 \times 0.3 \times \cos 60^0 = 0.009$ wb

iii) $\phi = 1 \times 0.06 \times 0.3 \times \cos 90^0 = 0$

Q.5. ***A small conducting loop of area 1 mm² is placed in the plane of a long straight wire at a distance of 20 cm from it. Current in the straight wire changes from 100 A to zero in 0.01 second. Find the average emf induced during this period.***

Solution :

Average emf $e = -\dfrac{\Delta\phi}{\Delta t} = \dfrac{\phi_1 - \phi_2}{\Delta t}$

Here ϕ_1 linked with the loop $= B.A. = \dfrac{\mu_0 I}{2\pi d}\ A$

$$= \frac{4\pi \times 10^{-7} \times 100}{2\pi \times 0.2} \times (10^{-6}) = 10^{-10}\ \text{Weber}$$

$\phi_2 =$ (final flux) $= 0.$

So, $e = \dfrac{10^{-10} - 0}{0.01} = 10^{-8}$ volt

Q.6. *The magnetic flux linked with the coil varies with time as $\phi = 3t^2 + 4t + 9$. Find the magnitude of induced e.m.f. When t=2sec.*

Solution:

$$\phi = 3t^2 + 4t + 9$$

$$e = \frac{d\phi}{dt} = \frac{d}{dt}(3t^2 + 4t + 9)$$

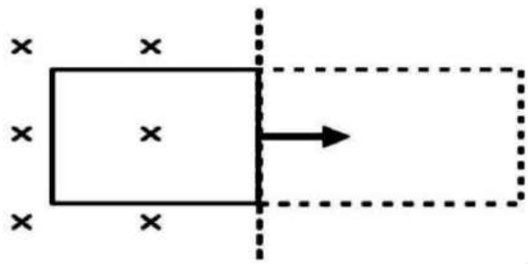
$$e = 6t + 4$$

At $t = 2s$; $e = 6 \times 2 + 4 = 16V$.

Q.7. *A magnetic field of induction 2T acts at right angles to a coil of area 100 cm^2 with 500 turns and having resistance of 10 Ω. If the coil is removed at a uniform rate from the field in 0.1 sec, find the*

(i) e.m.f. induced (ii) Current induced

(iii) Charge induced in the coil



Solution: (i) Induced e.m.f. $E = -N\frac{d\phi}{dt}$

$$E = N\left(\frac{\phi_1 - \phi_2}{t}\right)$$

$\phi_1 = nBA\cos\theta_1 = 2 \times 500 \times 100 \times 10^{-4} \cos(0°) = 10$

$[\theta_1 = 0°$, since area vector and B are in same direction$]$

$\phi_2 = BA\cos\theta_2 = 0$; $E = \frac{\phi_1 - \phi_2}{dt}$; $E = \frac{10 - 0}{0.1} = 100$

(ii) Induced current $I = \frac{E}{R} = \frac{100}{10} = 10A$

(iii) Induced charge $q = It = 10 \times 0.1 = 1C$

Q.8. The magnetic flux through a coil perpendicular to its plane is varying according to the relation $\phi_B = (5t^3 + 4t^2 + 2t - 5)$ weber. Calculate the induced current through the coil at $t = 2$ second. The resistance of the coil is 5Ω.

Solution: $\phi = 5t^3 + 4t^2 + 2t - 5$

$$|e| = \frac{d\phi}{dt} = 15t^2 + 8t + 2$$

at $t = 2\,\text{sec},\ = 15\times4 + 8\times2 + 2$

$$e = 78V\ ; \quad i = \frac{E}{R} = \frac{78}{5} = 15.6A$$

Q.9. A long solenoid with 1.5 turns per cm has a small loop of area $2.0\,cm^2$ placed inside the solenoid normal to its axis. If the current in the solenoid changes steadily from 2.0 A to 4.0 A in 1.0s. The emf induced in the loop is

Solution:

The magnetic field along the axis of solenoid is $B = \mu_0 ni$ where n is no. of turns per unit length.

flux hrough the smaller loop placed in solenoid is Since current in solenoid is changing, emf induced

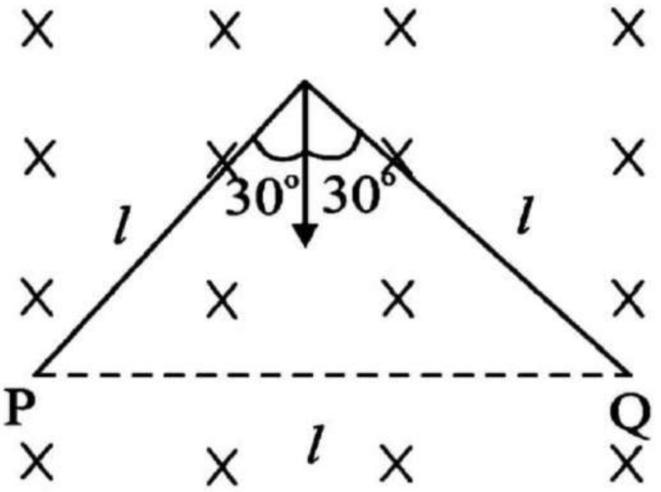
in loop is $e = \dfrac{d\phi}{dt} = \dfrac{d}{dt}[\mu_0 niA]$;

Arif Ansari 919726040154

$$= 4\pi\times10^{-7}\times1.5\times10^{2}\times2\times10^{-4}\times\left(\frac{4-2}{1-0}\right)$$

$$= 0.75\times10^{-7}\,V$$

Q.10. *A wire of length 2l is bent at mid point so that the angle between two halves is 60^0. If it moves as shown with a velocity v in a magnetic field B find the induced emf.*



Solution:

$e = Blv$. Here l = Effective length = PQ

$$\sin 30 = \frac{x}{l} \Rightarrow x = l \sin 30 = \frac{l}{2}$$

$$l_{eff} = 2x = 2\left(\frac{l}{2}\right) = l \quad e = B\,l_{eff}\,v = Blv$$

Q.11. *A copper rod of length 2m is rotated with a speed of 10 rps, in a uniform magnetic field of 1 tesla about a pivot at one end. The magnetic field is perpendicular to the plane of rotation.* Find the emf induced across its ends

Solution:

$$e = \frac{1}{2}B\omega l^2 = \frac{1}{2}B(2\pi n)l^2 = \pi Bnl^2$$

$$e = 3.14 \times 1 \times 10 \times 2 \times 2 = 125.6 \text{ volt}$$

Q.12. A 0.1 m long conductor carrying a current of 50 A is perpendicular to a magnetic field of 1.25 mT. Find the mechanical power to move the conductor with a speed of $1 ms^{-1}$

Solution:

Power $P=Fv$; $P=Bilv$; $l=0.1m$; $i=50$

$B=1.25\times10^{-3}$; $v=1m/sec$; $\therefore p=Bilv$

$=1.25\times10^{-3}\times50\times0.1\times1$; $=6.25\times10^{-3}$;

$$B=6.25\ mW$$

Q.13. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A=10cm^2$ and length = 20cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is $(\mu_0=4\pi\times10^{-7}TmA^{-1})$

Solution:

$$M=\frac{\mu_0 N_1 N_2 A}{L}$$

$$M=\frac{4\pi\times10^{-7}\times3\times10^2\times4\times10^2\times10^{-3}}{2\times10^{-1}}$$

$$=2.4\pi\times10^{-4}H$$

Q.14. The self-inductance of a coil having 200 turns is 10 milli henry. Calculate the magnetic flux through the cross-section of the coil corresponding to current of 4 milliampere. Also determine the total flux linked with each turn.

Solution:

Total magnetic flux linked with the coil,

$$N\phi = LI = 10^{-2} \times 4 \times 10^{-3} = 4 \times 10^{-5} Wb$$

$$\therefore \text{Flux per turn, } \phi = \frac{4 \times 10^{-5}}{200} = 2 \times 10^{-7} Wb$$

Q.15. Two solenoid A and B have same area of cross-section. The ratio of lengths of two is 1 : 2 and the ratio of number of turns is 2 : 1. Find the ratio of self inductance of A to that of B.

Solution :

Given $\frac{l_1}{l_2} = \frac{1}{2}$ *and* $\frac{N_1}{N_2} = 2$

Self inductance of solenoid is

$$L = \frac{\mu_0 N^2 A}{l} \quad \text{... (i)}$$

From (i) we can say

$$\frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2 \cdot \left(\frac{l_2}{l_1}\right) = (2)^2 \cdot (2) = 8.$$